INTRODUCTION TO MATHCAD

LESSON #2

INSERTING TEXT INTO MATHCAD. A useful feature of MATHCAD is the ability to insert text into any document for which calculations are being made. Text can be used to explain the principles behind the calculations, to annotate tables or for any purpose you desire. To insert text place the MATHCAD cross hair cursor at the location where text is to begin. To open a text window and begin inserting text, type the keystroke " , or use the Create Text option in the pull-down menu titled Text on the tool-bar. Simply type the text you desire and close the text window by clicking outside of the text window. The text can be moved about the document just as any other region that is defined by the definition of a variable or a calculation.

PLOTTING (CONTINUED!) We have observed how a graph can be created using the 'Create X-Y' option in the pull-down menu under GRAPHICS on the tool-bar, or by selecting the 'X-Y Plot' option from the Graphics pallette below the tool-bar. There are two important pre-requisites for a graph. First, you must define the range of the independent variable, say x, that is placed in the abscissa, and second, the independent variable, x, and the function f(x) to be plotted must be defined over a sufficient number of points to create a graph that appears smooth. MATHCAD draws a series of straight lines between the defined values of f(x) and x. If the number of points is insufficient in number the graph presents a crude appearance rather than that of a smooth line. For example, plot the function

$$f(x) = e^{-.25x} \sin(x)$$

over the range 2<=x<=10 with x defined as 2, 3, 4, 5, ...10. The result does not appear smooth. Now define x for the values 2, 2.2, 2.4, 2.6, 2.8,....10, i.e., in increments of 0.2. The result gives a much smoother appearance! Always keep these two important points in mind as you create a graph of a function.

You can also create a graph where the abscissa is not a simple independent variable. An example that illustrates this as well as the use of vectors rather than a range variable is the Lissajous figures. These are obtained by plotting $\cos(n\pi\theta)$ vs. $\sin(m\pi\theta)$, where n and m are integers. Do this by first defining a range variable,

i :1;50

and two vectors

x_i : sin(2π i/50) and y_i : cos(6π i/50)

Then open the graphics window and define x_i as the abscissa and y_i as the ordinate. This illustrates how a vector can be used in plotting. You can observe other Lissajous figures by changing the value of n and m! Define another vector, say z_i= cos(10π i/50) and plot this as well, creating two figures on one graphic window.

<u>LABELING GRAPHS</u>. By double-clicking on a graph you will open another window that has four tabs.....X-Y Axes, Traces, Labels and Defaults. Go to the Labels-tab. Its use is self-explanatory. Create a title for your Lissajous figures, and label the abscissa and ordinate sin $2\pi\theta$ and cos $6\pi\theta$, cos $10\pi\theta$ respectively. (Notice you can not retrieve Greek symbols into these labels.) If you have multiple traces, the default values for the appearance of these traces is given by the Traces-tab. If you desire to change the appearance of a trace from the default, you can do so using the settings of this tab. Make sure you understand the difference between the "Hide Arguments" and "Hide Legend" options.

<u>RESIZING GRAPHS</u>. The default size of a graphic window is relatively small. Sometimes this is sufficient; however, the window can be enlarged in either or both the height and width. Do this by creating a box about the graphic window. Notice that if you slide the mouse cursor down to the lower right-hand corner, a double-headed arrow appears. Use this to resize the graphic window in both height and width.

<u>SUBSCRIPTS WITH VECTORS & MATRICES</u> . We observed how to create vectors and matrices using the Matrices option under the Math pull-down menu or using the Vectors & Matrices pallette beneath the tool-bar. Subscript notation can also be useful in creating vectors and matrices and will often be useful in addressing individual elements of a vector or matrix. The subscript notation is called in MATHCAD by using the '[' keystroke or by using the X_i option from the pallette. In MATHCAD, type

x[2:5

then

x=

The result is a column vector with three elements whose values are 0, 0, and 5 respectively. First note that the subscript notation in MATHCAD for vectors and matrices always identifies the first element (row or column) as 0! Thus when you enter a value for x_2 and no values have been defined for x_0 and x_1, the values of '0' are inserted for x_0 and x_1 respectively. If you desired to define a vector whose components are 2, 3, 5 (x_0, x_1 , x_2 respectively), it could be accomplished by typing

x[0:2 x[1:3 x[2:5

or it could be defined by inserting the specific values into place-holders of the 3x1 matrix created with the define matrix option of the matrix pallette. Generally speaking, the subscript approach is more cumbersome when defining a full matrix, but the subscript notation is more efficient when defining a specific element.

Matrices are defined using two subscripts. Thus the MATHCAD code

A[2,2:5

defines a 3x3 matrix whose elements $A_{i,j}$ are each zero except for $A_{2,2} = 5$.

The numeration of the first element in a vector or the first row and first column of a matrix can be changed from the default value of zero. The MATHCAD command

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changes the value of the initial element to one (1). Other constant integer values can be used in this command; however, a variable can not be used.

Matrix algebra is often used to solve simultaneous linear equations. Consider the set of four equations:

$$10a + 3b - 4d = 12$$
$$a + 3b + 2c - d = -4$$
$$2b - 4c + 3d = 6$$
$$3a - 2b - c = 8$$

These equations can be formatted in matrix notation as

$$B*X = Y$$

where

$$B = \begin{bmatrix} 10 & 3 & 0 & -4 \\ 1 & 3 & 2 & -1 \\ 0 & 2 & -4 & 3 \\ 3 & -2 & -1 & 0 \end{bmatrix} \qquad Y = \begin{bmatrix} 12 \\ -4 \\ 6 \\ 8 \end{bmatrix}$$

The solution for X is

$$X = B^{-1} * Y$$

where B^{-1} is the inverse of the coefficient matrix, B. Solve these simultaneous equations in MATHCAD and check your solution by carrying out the matrix multiplication B*X after you obtain the solution vector, X using the matrix inverse.

<u>SOLVING ALGEBRAIC EQUATIONS</u>. Algebraic equations of one variable or a system of n equations can be solved conveniently in MATHCAD. Of course, if the equations are linear, the system of linear equations can be solved by matrix algebra after defining the coefficient matrix and the column vector of non-homogeneous terms. We will examined this above. An equation with a single variable is solved using a root-finding algorithm known as the secant method. In this method an initial guess is required and the algorithm iterates from this guess until a true root is found. A warning is given if no root is found. First the equation must be formatted in the

manner, $f(x) = 0$. Thus an equation

$$x^3 = e^x$$

must be written in the format

$$x^3 - e^x \qquad \text{or} \qquad f(x) = x^3 - e^x$$

Let's examine f(x) and determine the real roots, which are the values of x that satisfy the equation

$$f(x) = 0 = x^3 - e^x$$

Because this is a cubic equation, we may anticipate multiple roots. However, this is not a polynomial, so the number of real roots is not clear and some of the roots may be imaginary. In any event the root-finding algorithm will determine each root singularly or one root for any execution. How can we ensure that distinctly different roots will be found? Generally, before beginning root finding it is desirable to have some idea of what the function "looks like" when plotted. From observing the plot, more reasonable estimates of guesses for the value of the root can be made. <u>Begin by plotting the function $f = x^3 - e^x$ vs x over the range -10<=x<=10</u>. To observe where the function crosses the axes, f = 0, it will be useful to select the "Crossed" option under "Axes Style". It does appear that the function crosses from below the y=0 axis and subsequently crosses again into negative y region, indicating two real roots. You can find the roots using the 'root' function as follows. First, define a guess value of a root (say x=-5), then use the root function.

x:-5
a:root(x^3 - e^x , x)
a=

The result should be a=1.857. Try different initial guesses. You should be able to find a second root at x=4.536.

These roots may be more readily observed in the graphing window if you change the limits on the abscissa. This is accomplished as follows. Click on the graph to open the border window about the graph. Now click on the numerical value of the upper limit on the ordinate. Change this to 10. Repeat this for the lower limit, changing this limit to -10. Now click outside of the blue border and a new graph is created that enlarges greatly the area in which the roots occur. This techniques is often useful in looking for candidate guesses for roots.

Now find the roots to the polynomial

$$g(z) = z^3 -10z + 2$$

Approaching the problem as described above you should find three real roots (-3.258, .201, and 3.057). Another function exists for root-finding when the function is a polynomial such as this one. Define a column vector whose coefficients are the coefficients of the polynomial, beginning with the coefficient of the zero-order term in the first element and the coefficient of the highest order term in the last element. For this polynomial, the elements are (in order, lowest to highest) 2, -10, 0, 1. Type

coef[0:2 coef[1:-10 coef[2:0 coef[3:1

Then use the MATHCAD function polyroots(coef) where coef is the vector of the coefficients.

polyroots(coef)=

The function returns the roots. Try this for the function g(z) described above and obtain the same roots noted above.

<u>SOLVING NON-LINEAR SIMULTANEOUS EQUATIONS</u> . The 'root ' function works well for a single equation, f(x). When two are more equations exist, the solution is found using the solve-block defined by the Given-Find entries. Consider the equations

$$y = 6 - 2e^{-.5x}$$
$$y = 4.5 + .2x^{1.2}$$

The first step is to define guess values for x and y, and to do this it may be desirable to plot the functions and observe approximately where the solution roots exist. [HINT: Define x over the range $0 \le x \le 10$ and plot the two equations for y above. Note that in order to plot two functions on a single plot you must identify two distinct functions, say y1 and y2.] Define your guess values. For example,

x:1 y:1

Now type

Given

This opens the Solve-block. Now, below the Given (within the Solve-block) enter the equations above USING [Ctrl-=] FOR THE '=' SIGN. The [Ctrl-=] keystroke defines an equality within the Solve-block. Complete or close the Solve-block by typing

Find(x,y)=

The x-y values for one pair of the roots will be found. To find the other pair of roots, you must use different initial values (guesses) for x,y. [*Ans:(.786, 4.65);* (4.82, 5.82)]